Introduction to Languages for Scientific Computing, winter semester 14/15: Final Exam

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1) Floating point arithmetic

1.1) [6 points]

Consider the following floating point arithmetic with normalization:

$$\beta = 2, \quad t = 4, \quad e_{min} = -5, \quad e_{max} = 5.$$

1. What is $a = fl(12.34)$?
2. What is the absolute and relative representation error of a?
3. What is $b = fl(0.321)$?
4. What is the absolute and relative representation error of b?
5. What is $c = fl(a + b)$?
6. What is the absolute and relative representation error of c?

(One point each).

1.2) [2 points]

Consider single and double precision arithmetic with normalization:

$$\beta = 2, \quad t = 24, \quad e_{min} = -125, \quad e_{max} = 128,$$
$$\beta = 2, \quad t = 53, \quad e_{min} = -1021, \quad e_{max} = 1024.$$

How many double precision numbers are between two consecutive single precision numbers (i.e. how many d_js in $]s_i, s_{i+1}[$)?

2) Matlab

2.1) [2 points]

What is the output of the following program?

```
A=[2 0;0 2];
B=[1 2;-3 4];
M = A'*B
v([4 3 1 2]) = [M(1,:),M(2:)]
```

2.2) [3 points]

What is the output of the following program?

1. `sin(pi)`
2. `2^54 + 2 == 2^54`
3. `r = rand; s = single(r); t = double(s); s == t`

2.3) [4 points]

Write a function that takes a vector of size $3n$ and returns

$$\begin{bmatrix} I_n & I_n & 0 \\ -I_n & 0 & I_n \\ I_n & -I_n & 0 \end{bmatrix} \cdot v$$

without constructing the matrix, or I_n.

2.4) [3 points]

Write a recursive program that constructs H_n:

$$H_n = \begin{bmatrix} n & n & n & n & n & n \\ n & n-1 & n-1 & \dots & n-1 & n-1 \\ n & n-1 & n-2 & \dots & n-2 & n-2 \\ n & n-1 & n-2 & \dots & \vdots & \vdots \\ n & n-1 & n-2 & \dots & 1 & 1 \\ n & n-1 & n-2 & \dots & 1 & 0 \end{bmatrix}$$

3) Mathematica

The constructs `For, While, Do, Table` and `If` are forbidden.

3.1) [2 points]

Explain the following expression

```
f[x__,n_Integer];/Length[{x}]>n -> g[x]
```

3.2) [3 points]

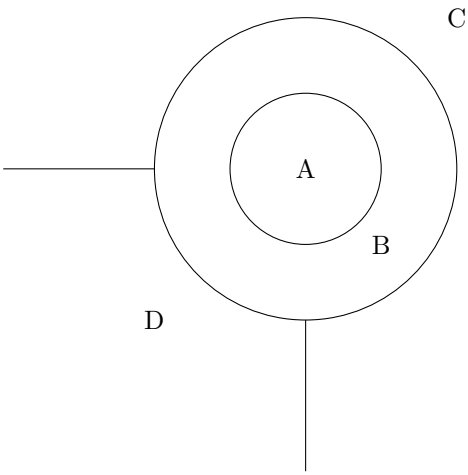
What is the output of the following program?

```
Map[ {#[[1]][#[[2]]], #[[1]][2]#[[2]], #[[2]]#[[1]][#[[1]][1]]},
     {{#^2-1&, 2},{(#-1)(#+1)&, -2}}
  ]
```

3.3) [4 points]

Write a function `H[i,j,n]` that returns the (i, j)-entries of H_n as in 2.4 with bound checks. If out-of-bounds or no integer: return "NaN".

3.4) [2 points]



Function ABC as follows:

- in A (circle size 1 around (0,0), incl. borders): $x^2 + y^2$
- in B (circle size 2 around that, incl. outer borders):cos $\sqrt{x^2 + y^2}$
- in C (outside circle): 0
- in D (outside circle, lower left quarter): π

Define function ABC.

3.5) [5 points]

Captian Kirk wants to find the roots of $\left|\sin\left(x^2\right) - .2\right| - .3$ in $[0, \pi]$. FindRoots takes an equation and a tuple of the form {`variable, initial_guess`} and returns one solution in the form {`variable->value`}. Help him follow these steps:

1. Define f.
2. Apply FindRoots for initial guesses given by `Table[i,{i,-.2,4,.2}]`.
3. Filter those results outside the range.
4. Merge results which are "almost equal".
5. Plot the function as shown in figure ... (Function + Dots at every root).

4) C

4.1) [5 points]

Leia successfully compiles the following program:

```
#include <stdlib.h>
#include <stdio.h>

void function(int* arr, int n){
  int idx, current,i,j,tmp;

  for (i = 0; i < n; i++) {
    current = -1;
    idx = -1;

    for(j=i; j < n; j++) {
      if (arr[j] > current) {
        current = arr[j];
        idx = j;
      }
    }

    printf("iter:%d\tidx:%d\n", i, idx);

    tmp = arr[idx];
    arr[idx] = arr[i];
    arr[i] = tmp;
  }
}
```

```
int main (int argc, char* argv[]) {
  int i, n = 6;

  int* arr = (int*) malloc(n*sizeof(int));

  for (i=0; i < n; i++) {
    arr[i] = (7*i+2) % 5;
  }

  function(arr, n);

  printf("[");

  for (i = 0; i < n; i++) {
    printf("%d␣", arr[i]);
  }

  printf("]");
  return 0;
}
```

1. What is the output of the program?
2. What is generally computed by function?
3. If `for (i = 0; i < n; i++)` is replaced with `for (i = 0; i < n-1; i++)`, would the output differ? Why?
4. If `for (j = i; j < n; j++)` is replaced with `for (j = i+1; j < n; j++)`, would the output differ? Why?

4.2) [3 points]

The Princess wants to make calculations with the class of matrices P_n. Construct lower-triangular matrix $P(i,j) = \frac{i+j}{ij+1}$ for $i < j$, 0 else. Ensure that it is stored by columns.

```
#include <stdlib.h>
#include <stdio.h>

int main(int argc, char* argv[]) {
  int n;
  if(argc >1) n=atoi(argv[1]);

  //your code
}
```