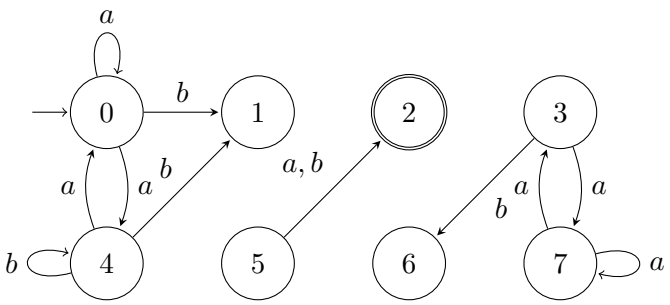
Advanced Automata Theory 1st Exam

14^{th} August 2018

Exercise 1 - Minimizing NTAs

An automata $\mathcal{A}$ with 8 states is given.



a) Give a winning strategy for the spoiler or duplicator for the game $BG(\mathcal{A}, 1, \mathcal{A}, 4)$.

b) Give a winning strategy for the spoiler or duplicator for the game $BG(\mathcal{A}, 3, \mathcal{A}, 0)$.

c) Give two minimal NFAs $\mathcal{A}_1 \neq \mathcal{A}_\in$ with $L(\mathcal{A}_1) = L(\mathcal{A}_2)$.

Exercise 2 - Learning DFAs

A sampleset $S = (S_+, S_-)$ is given.

a) Give a 3 state DFA which accepts S.

b) Apply the RPNI-algorithm on the sampleset S.

Exercise 3 - Logic on words

$\Sigma = \{a, b, c\}$.

a) Give an LTL sentence recognizing

$$L_1 = \{w \in \Sigma^* \mid \text{the infix } ba \text{ only appears once }\}.$$

b) Give an FO sentence recognizing L_1.

c) Give an MSO sentence recognizing

$$L_2 = L((\Sigma c)^+).$$

Exercise 4 - Tree languages

$\Sigma = \Sigma_0 \cup \Sigma_2$, $\Sigma_0 = \{a, b, c\}, \Sigma_2 = \{f\}$.

a) Give a NTA recognizing

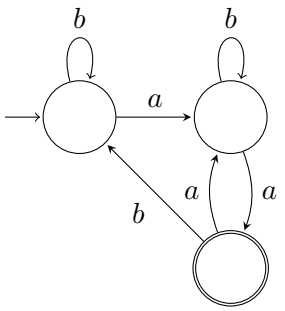
$$T = \{t \in T_\Sigma \mid ab \text{ is in yield}(w)\}$$

b) Give a NUTA recognizig:

$$T = \{t \in T_\Sigma \mid t \text{ has a path with at least one } b \text{ and no } a\text{'s } \}$$

Exercise 5 - Transition Monoid

An automata $\mathcal{A}$ with 3 states is given.



a) Give a graphical representation of the transition monoid of $\mathcal{A}$ $M(\mathcal{A})$.

b) Give a non-trivial subgroup in $M(\mathcal{A})$ by listing the elements and identifying the neutral element.

c) Is there a star-free expression recognizing $L(\mathcal{A})$?

Exercise 6 - Pushdown-Automata

A pushdown system $\mathcal{P}$ and a configuration set C is given.

a) Present a P-NFA recognizing C.

b) Apply the saturation algorithm from the lecture to construct a P-NFA recognizing $pre^*_P(C)$. For each of the five transitions that are added, write down that transition and the corresponding pushdown rule.

c) Does $p_0ba \in pre^*_P(C)$ hold?

Exercise 7 - Logic on trees

Give an MSO sentence which recognizes (??)

Exercise 8 - 2-Register-Machine Reduction

Consider the modified 2-Register Machines which have no $DEC(X_i)$. Show that the halting problem for these machines is decidable by giving a reduction to the emptiness problem of NFAs.